4D Würfel

Michael Nüsken, Paderborn
10. Februar 2003

Wir verwenden das Paket convex von Matthias Franz, Paderborn/Grenoble, um konvexe Hüllen auszurechnen und zu zeichnen.

```
> restart:
  with(LinearAlgebra):
  changecoords:='changecoords': with(plots):
  read 'convex-1.0/convex.m';
  polar:='polar': with(convex):
convex 1.0 as of Fri Jan 24 15:07:34 CET 2003
This version is still UNDER CONSTRUCTION!
```

Diese Prozedur berechnet ein auf rationale Koordinaten gerundetes orthonormiertes Koordinatensystem, dessen letzter Vektor der gegebene ist.

```
> hyperplanesystem:=proc(normalvector)
    local n;
    n:=Vector(normalvector);
    LinearAlgebra:-NullSpace(Transpose(Matrix(n)));
    LinearAlgebra:-GramSchmidt(%,normalized);
    [op(%),expand(n/Norm(n,2))];
    Matrix(%);
    Map( convert, evalf(%), rational );
  end proc:
```

Da die Prozedur leicht nicht deterministisch ist, geben wir hier ein schönes Ergebnis vor.

```
> hyperplanesystem([1,1,1,1]):=1/2*<<-1,1,1,-1>|<1,-1,1,-1>|<1,1,-
  1,-1>|<1,1,1,1>>;
  % . Transpose(%), Transpose(%).%;
```

$$\text{hyperplanesystem}([1, 1, 1, 1]) := \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Wir brauchen natürlich eine Rechendarstellung des vierdimensionalen Würfels. Das erledigt die Prozedur `cube` aus dem Paket convex.

```
> C:=cube(4):
```

Diese Zeilen haben mir geholfen, die Syntax des convex Paketes zu ergründen, und für unsere Zwecke nutzbar zu machen.

```
Digits:=10:
```

```
Hpoint:=1/2*<1,1,1,1>:
Hbasis:=hyperplanesystem( Hpoint ):
convert(Transpose(%),listlist):
map(Norm,map(Vector,%),2): evalf(%): evalf(%,8);
Hinv:=IdentityMatrix(3,4).Hbasis^(-1);
H:=convhull( Hpoint, op(map(line,
convert(Transpose(Hbasis[1..-1,1..-2]),listlist) )) );
CS:=image( intersection( H, C4 ), Hinv, Hinv.Hpoint );
plots[display]( draw(CS), scaling=constrained);
```

Ein Film besteht aus einzelnen Bildern. Hier wird ein solches erzeugt.

```
> polyfilmpic:=proc(
      P::list, C::list,
      H::Matrix, h::Vector
    )::PLOT3D;
    local Q, _H, W;
    Q:=IdentityMatrix(3,4) . H^(-1);
    _H:=convhull( convert(h,list), op(map(line,
  convert(Transpose(H[1..-1,1..-2]),listlist) )) );
    if Wireframe then
      P;
      map( image, %, Q, -Q.h );
      zip( (x,y)->display(draw(x),color=y,style=wireframe), %, C
  );
      W:=display( %, args[5..-1] );
    else W:=[];
    end if;
    map( intersection, P, _H );
    map( image, %, Q, -Q.h );
    zip( (x,y)->display(draw(x),color=y), %, C );
    display( [op(W), op(%)], args[5..-1] );
  end proc:
> polyfilm:=proc(
      P,
      H::procedure, h::procedure,
      L::list
    )::PLOT3D;
    local Pfacets,n, Pcolors,F,z,st;
    # Pfacets:=[P]; #
    Pfacets:=map( convert, facets(P), POLYHEDRON );
    n:=nops(Pfacets);
    Pcolors:=[seq( COLOR(HUE,i/n), i=0..n-1 ) ];
    F:=NULL;
    for z in L do
      userinfo(2,polyfilm,‘Computing picture ‘,z);
      st:=time();
      F:=F,polyfilmpic( Pfacets, Pcolors,
  convert(evalf(H(z)),rational), convert(evalf(h(z)),rational) );
      time()-st;
      userinfo(2,polyfilm,nprintf("Used %a seconds.",%));
    end do;
    display( [F], scaling=constrained, insequence=true );
  end proc:
> intervallpartition:=proc( r::range, N::posint )::list;
```

```
    local L, x;
    L:=NULL;
    for x from lhs(r) to rhs(r)-(rhs(r)-lhs(r))/(2*N)
      by (rhs(r)-lhs(r))/N do
      L:=L,x;
    end do;
    [L];
  end proc:
> polyfilm_orthorotation:=proc(
      P,
      n, alpha, beta, gamma, delta,
      r::range, N::posint
    )::PLOT3D;
    local z,e1,e2,e3,e4,H,c,s,h,i,UNAPPLY;
    e1:=<1,0,0,0>;
    e2:=<0,1,0,0>;
    e3:=<0,0,1,0>;
    e4:=<0,0,0,1>;
    H:=hyperplanesystem(n);
    c:=cos(2*Pi*gamma(z));
    s:=sin(2*Pi*gamma(z));
    H:=Matrix(map(simplify,[c*e1+s*e2,-s*e1+c*e2,e3,e4])) . H;
    c:=cos(2*Pi*beta(z));
    s:=sin(2*Pi*beta(z));
    H:=Matrix(map(simplify,[c*e1+s*e3,e2,-s*e1+c*e3,e4])). H;
    c:=cos(2*Pi*alpha(z));
    s:=sin(2*Pi*alpha(z));
    H:=Matrix(map(simplify,[e1,c*e2+s*e3,-s*e2+c*e3,e4])) . H;
    # H:=Matrix(map(simplify,[c*e1-s*e4,e2,e3,s*e1+c*e4])) . H;
    H:=simplify(combine(expand(H),trig));
    h:=simplify(delta(z) * H . e4); if h=0 then h:=<0,0,0,0>; end
  if;
    UNAPPLY:=proc(ex,z)
      codegen[optimize](unapply(ex,z));
    end proc:
    'polyfilm'( P,
       'UNAPPLY'(H,z),
       'UNAPPLY'(h,z),
       'intervallpartition'( r, N ) );
  end proc:
printlevel:=1:
h:=<1,1,1,1>:
H:=hyperplanesystem(h):
'polyfilm'( C4, 'unapply'(H,z), 'unapply'(simplify(z*h),z),
intervallpartition( -1..1, 80 ) );
> infolevel:='infolevel':
  # infolevel[polyfilm]:=2:
> Wireframe:=false:
  film1:=polyfilm_orthorotation(
    C, [1,1,1,1],
    z->0, z->0, z->0,
    z->z,
    -2..2, 256
```

```
);
st:=time():
film1:=eval(film1):
time()-st;
```

$$film1 := \text{polyfilm}\left(\text{POLYTOPE}(4, 4, 16, 8), \text{UNAPPLY}\left(\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}, z\right), \text{UNAPPLY}\left(\begin{bmatrix} \frac{z}{2} \\ \frac{z}{2} \\ \frac{z}{2} \\ \frac{z}{2} \end{bmatrix}, z\right), \text{intervallpartition}(-2 .. 2, 256)\right)$$

51.164

```
> Wireframe:=false:
film6:=polyfilm_orthorotation(
  C, [1,1,1,1],
  z->z, z->0, z->0,
  z->0,
  0..1, 256
);
st:=time():
film6:=eval(film6):
time()-st;
```

$$film6 := \text{polyfilm}\left(\text{POLYTOPE}(4, 4, 16, 8), \text{UNAPPLY}\left(\right.\right.$$

$$\left[-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right]$$

$$\left[\frac{1}{2}\cos(2\pi z) - \frac{1}{2}\sin(2\pi z), -\frac{1}{2}\cos(2\pi z) - \frac{1}{2}\sin(2\pi z), \frac{1}{2}\cos(2\pi z) + \frac{1}{2}\sin(2\pi z),\right.$$

$$\frac{1}{2}\cos(2\pi z)-\frac{1}{2}\sin(2\pi z)\Big]$$

$$\Big[\frac{1}{2}\cos(2\pi z)+\frac{1}{2}\sin(2\pi z),\ \frac{1}{2}\cos(2\pi z)-\frac{1}{2}\sin(2\pi z),\ \frac{1}{2}\sin(2\pi z)-\frac{1}{2}\cos(2\pi z),$$

$$\frac{1}{2}\cos(2\pi z)+\frac{1}{2}\sin(2\pi z)\Big]$$

$$\Big[\frac{-1}{2},\frac{-1}{2},\frac{-1}{2},\frac{1}{2}\Big],z\Big),\ \mathrm{UNAPPLY}\left(\begin{bmatrix}0\\0\\0\\0\end{bmatrix},z\right),\ \mathrm{intervallpartition}(0\ ..\ 1,\ 256)\Big)$$

81.207

```
> Wireframe:=false:
  film7:=polyfilm_orthorotation(
    C, [1,1,1,1],
    z->z, z->0, z->0,
    z->1/3,
    0..1, 256
  );
  st:=time():
  film7:=eval(film7):
  time()-st;
```

$$film7 := \mathrm{polyfilm}\Big(\mathrm{POLYTOPE}(4,\ 4,\ 16,\ 8),\ \mathrm{UNAPPLY}\Big($$

$$\Big[\frac{-1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\Big]$$

$$\Big[\frac{1}{2}\cos(2\pi z)-\frac{1}{2}\sin(2\pi z),\ -\frac{1}{2}\cos(2\pi z)-\frac{1}{2}\sin(2\pi z),\ \frac{1}{2}\cos(2\pi z)+\frac{1}{2}\sin(2\pi z),$$

$$\frac{1}{2}\cos(2\pi z)-\frac{1}{2}\sin(2\pi z)\Big]$$

$$\Big[\frac{1}{2}\cos(2\pi z)+\frac{1}{2}\sin(2\pi z),\ \frac{1}{2}\cos(2\pi z)-\frac{1}{2}\sin(2\pi z),\ \frac{1}{2}\sin(2\pi z)-\frac{1}{2}\cos(2\pi z),$$

$$\frac{1}{2}\cos(2\pi z)+\frac{1}{2}\sin(2\pi z)\Big]$$

$$\left[\left[-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right],z\right],\ \text{UNAPPLY}\left(\left[\begin{array}{c}\frac{1}{6}\\ \frac{1}{6}\cos(2\pi z)-\frac{1}{6}\sin(2\pi z)\\ \frac{1}{6}\cos(2\pi z)+\frac{1}{6}\sin(2\pi z)\\ \frac{1}{6}\end{array}\right],z\right),\ \text{intervallpartition}(0\ ..\ 1,256)\right)$$

102.507

```
> Wireframe:=false:
  film8:=polyfilm_orthorotation(
    C, [1,1,1,1],
    z->z, z->z, z->z,
    z->0,
    0..1, 256
  );
  st:=time():
  film8:=eval(film8):
  time()-st;
```

$$film8 := \text{polyfilm}\left(\text{POLYTOPE}(4,4,16,8),\ \text{UNAPPLY}\left(\right.\right.$$

$$\left[-\frac{1}{4}\cos(4\pi z)-\frac{1}{4}-\frac{1}{4}\sin(4\pi z)-\frac{1}{2}\sin(2\pi z),\ \frac{1}{4}\cos(4\pi z)+\frac{1}{4}+\frac{1}{4}\sin(4\pi z)-\frac{1}{2}\sin(2\pi z),\right.$$
$$\left.\frac{1}{4}\cos(4\pi z)+\frac{1}{4}-\frac{1}{4}\sin(4\pi z)+\frac{1}{2}\sin(2\pi z),\ \frac{1}{4}\cos(4\pi z)+\frac{1}{4}-\frac{1}{4}\sin(4\pi z)-\frac{1}{2}\sin(2\pi z)\right]$$

$$\left[\frac{1}{4}\cos(4\pi z)+\frac{1}{4}-\frac{1}{2}\sin(4\pi z)+\frac{1}{8}\cos(2\pi z)-\frac{1}{8}\cos(6\pi z)+\frac{3}{8}\sin(2\pi z)-\frac{1}{8}\sin(6\pi z),\right.$$
$$-\frac{1}{4}\cos(4\pi z)-\frac{1}{4}-\frac{1}{8}\cos(2\pi z)+\frac{1}{8}\cos(6\pi z)-\frac{3}{8}\sin(2\pi z)+\frac{1}{8}\sin(6\pi z),$$
$$\frac{1}{4}\cos(4\pi z)+\frac{1}{4}+\frac{1}{2}\sin(4\pi z)-\frac{1}{8}\cos(2\pi z)+\frac{1}{8}\cos(6\pi z)+\frac{3}{8}\sin(2\pi z)-\frac{1}{8}\sin(6\pi z),$$
$$\left.\frac{1}{4}\cos(4\pi z)+\frac{1}{4}-\frac{1}{8}\cos(2\pi z)+\frac{1}{8}\cos(6\pi z)+\frac{3}{8}\sin(2\pi z)-\frac{1}{8}\sin(6\pi z)\right]$$

$$\left[\frac{1}{4}\sin(4\pi z)+\frac{1}{2}\cos(4\pi z)-\frac{1}{8}\sin(6\pi z)-\frac{1}{8}\sin(2\pi z)-\frac{1}{8}\cos(2\pi z)+\frac{1}{8}\cos(6\pi z),\right.$$
$$-\frac{1}{4}\sin(4\pi z)+\frac{1}{2}+\frac{1}{8}\sin(6\pi z)+\frac{1}{8}\sin(2\pi z)+\frac{1}{8}\cos(2\pi z)-\frac{1}{8}\cos(6\pi z),$$

$$\frac{1}{4}\sin(4\pi z)-\frac{1}{2}\cos(4\pi z)+\frac{1}{8}\sin(6\pi z)+\frac{1}{8}\sin(2\pi z)-\frac{1}{8}\cos(2\pi z)+\frac{1}{8}\cos(6\pi z),$$

$$\frac{1}{4}\sin(4\pi z)+\frac{1}{2}+\frac{1}{8}\sin(6\pi z)+\frac{1}{8}\sin(2\pi z)-\frac{1}{8}\cos(2\pi z)+\frac{1}{8}\cos(6\pi z)\Bigg]$$

$$\left[\frac{-1}{2},\frac{-1}{2},\frac{-1}{2},\frac{1}{2}\right],z\Bigg)\,\mathrm{UNAPPLY}\left(\begin{bmatrix}0\\0\\0\\0\end{bmatrix},z\right),\mathrm{intervallpartition}(0\ldots1,256)\Bigg)$$

170.274

```
> plotsetup(window):
  display(film1,insequence=true,title="Film 1: Wanderung längs
  <1,1,1,1>");
  plotsetup(default):
> plotsetup(window):
  display(film6,insequence=true,title="Film 6: Drehung in x',y'");
  plotsetup(default):
> plotsetup(window):
  display(film7,insequence=true,title="Film 7: Drehung in x',y'
  mit 1/3 Verschiebung nach außen");
  plotsetup(default):
> plotsetup(window):
  display(film8,insequence=true,title="Film 8: Drehung
  alpha=beta=gamma=z");
  plotsetup(default):
```

```
> Wireframe:=true:
  film1a:=polyfilm_orthorotation(
    C, [1,1,1,1],
    z->0, z->0, z->0,
    z->z,
    -2..2, 256
  );
  st:=time():
  film1a:=eval(film1a):
  time()-st;
```

$$film1a := \text{polyfilm}\left(\begin{array}{l} \text{POLYTOPE}(4, 4, 16, 8), \\ \text{UNAPPLY}\left(\begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{1}{2} \end{bmatrix}, z\right), \text{UNAPPLY}\left(\begin{bmatrix} \frac{z}{2} \\ \frac{z}{2} \\ \frac{z}{2} \\ \frac{z}{2} \end{bmatrix}, z\right), \\ \text{intervallpartition}(-2 .. 2, 256) \end{array}\right)$$

185.597

```
> Wireframe:=true:
  film6a:=polyfilm_orthorotation(
    C, [1,1,1,1],
```

```
    z->z, z->0, z->0,
    z->0,
    0..1, 256
  );
  st:=time():
  film6a:=eval(film6a):
  time()-st;
```

$$film6a := \text{polyfilm}\left(\text{POLYTOPE}(4, 4, 16, 8), \text{UNAPPLY}\left(\right.\right.$$

$$\left[\frac{-1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right]$$

$$\left[\frac{1}{2}\cos(2\pi z) - \frac{1}{2}\sin(2\pi z), -\frac{1}{2}\cos(2\pi z) - \frac{1}{2}\sin(2\pi z), \frac{1}{2}\cos(2\pi z) + \frac{1}{2}\sin(2\pi z),\right.$$
$$\left.\frac{1}{2}\cos(2\pi z) - \frac{1}{2}\sin(2\pi z)\right]$$

$$\left[\frac{1}{2}\cos(2\pi z) + \frac{1}{2}\sin(2\pi z), \frac{1}{2}\cos(2\pi z) - \frac{1}{2}\sin(2\pi z), \frac{1}{2}\sin(2\pi z) - \frac{1}{2}\cos(2\pi z),\right.$$
$$\left.\frac{1}{2}\cos(2\pi z) + \frac{1}{2}\sin(2\pi z)\right]$$

$$\left.\left[\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{1}{2}\right], z\right), \text{UNAPPLY}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, z\right), \text{intervallpartition}(0 .. 1, 256)\right)$$

296.616

```
> Wireframe:=true:
  film7a:=polyfilm_orthorotation(
    C, [1,1,1,1],
    z->z, z->0, z->0,
    z->1/3,
    0..1, 256
  );
  st:=time():
  film7a:=eval(film7a):
  time()-st;
```

$$film7a := \text{polyfilm}\left(\text{POLYTOPE}(4, 4, 16, 8), \text{UNAPPLY}\left(\right.\right.$$

$$\left[\frac{-1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right]$$

$$\left[\frac{1}{2}\cos(2\pi z) - \frac{1}{2}\sin(2\pi z), -\frac{1}{2}\cos(2\pi z) - \frac{1}{2}\sin(2\pi z), \frac{1}{2}\cos(2\pi z) + \frac{1}{2}\sin(2\pi z),\right.$$
$$\left.\frac{1}{2}\cos(2\pi z) - \frac{1}{2}\sin(2\pi z)\right]$$

$$\left[\frac{1}{2}\cos(2\pi z) + \frac{1}{2}\sin(2\pi z), \frac{1}{2}\cos(2\pi z) - \frac{1}{2}\sin(2\pi z), \frac{1}{2}\sin(2\pi z) - \frac{1}{2}\cos(2\pi z),\right.$$
$$\left.\frac{1}{2}\cos(2\pi z) + \frac{1}{2}\sin(2\pi z)\right]$$

$$\left.\left[\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{1}{2}\right], z\right), \text{UNAPPLY}\left(\begin{bmatrix} \frac{1}{6} \\ \frac{1}{6}\cos(2\pi z) - \frac{1}{6}\sin(2\pi z) \\ \frac{1}{6}\cos(2\pi z) + \frac{1}{6}\sin(2\pi z) \\ \frac{1}{6} \end{bmatrix}, z\right), \text{intervallpartition}(0 .. 1, 256)\right)$$

318.779

```
> Wireframe:=true:
  film8a:=polyfilm_orthorotation(
    C, [1,1,1,1],
    z->z, z->z, z->z,
    z->0,
    0..1, 256
  );
  st:=time():
  film8a:=eval(film8a):
  time()-st;
```

$$film8a := \text{polyfilm}\left(\text{POLYTOPE}(4, 4, 16, 8), \text{UNAPPLY}\left(\right.\right.$$

$$\left[-\frac{1}{4}\cos(4\pi z)-\frac{1}{4}-\frac{1}{4}\sin(4\pi z)-\frac{1}{2}\sin(2\pi z), \frac{1}{4}\cos(4\pi z)+\frac{1}{4}+\frac{1}{4}\sin(4\pi z)-\frac{1}{2}\sin(2\pi z),\right.$$
$$\left.\frac{1}{4}\cos(4\pi z)+\frac{1}{4}-\frac{1}{4}\sin(4\pi z)+\frac{1}{2}\sin(2\pi z), \frac{1}{4}\cos(4\pi z)+\frac{1}{4}-\frac{1}{4}\sin(4\pi z)-\frac{1}{2}\sin(2\pi z)\right]$$

$$\left[\frac{1}{4}\cos(4\pi z)+\frac{1}{4}-\frac{1}{2}\sin(4\pi z)+\frac{1}{8}\cos(2\pi z)-\frac{1}{8}\cos(6\pi z)+\frac{3}{8}\sin(2\pi z)-\frac{1}{8}\sin(6\pi z),\right.$$
$$-\frac{1}{4}\cos(4\pi z)-\frac{1}{4}-\frac{1}{8}\cos(2\pi z)+\frac{1}{8}\cos(6\pi z)-\frac{3}{8}\sin(2\pi z)+\frac{1}{8}\sin(6\pi z),$$
$$\frac{1}{4}\cos(4\pi z)+\frac{1}{4}+\frac{1}{2}\sin(4\pi z)-\frac{1}{8}\cos(2\pi z)+\frac{1}{8}\cos(6\pi z)+\frac{3}{8}\sin(2\pi z)-\frac{1}{8}\sin(6\pi z),$$
$$\left.\frac{1}{4}\cos(4\pi z)+\frac{1}{4}-\frac{1}{8}\cos(2\pi z)+\frac{1}{8}\cos(6\pi z)+\frac{3}{8}\sin(2\pi z)-\frac{1}{8}\sin(6\pi z)\right]$$

$$\left[\frac{1}{4}\sin(4\pi z)+\frac{1}{2}\cos(4\pi z)-\frac{1}{8}\sin(6\pi z)-\frac{1}{8}\sin(2\pi z)-\frac{1}{8}\cos(2\pi z)+\frac{1}{8}\cos(6\pi z),\right.$$
$$-\frac{1}{4}\sin(4\pi z)+\frac{1}{2}+\frac{1}{8}\sin(6\pi z)+\frac{1}{8}\sin(2\pi z)+\frac{1}{8}\cos(2\pi z)-\frac{1}{8}\cos(6\pi z),$$
$$\frac{1}{4}\sin(4\pi z)-\frac{1}{2}\cos(4\pi z)+\frac{1}{8}\sin(6\pi z)+\frac{1}{8}\sin(2\pi z)-\frac{1}{8}\cos(2\pi z)+\frac{1}{8}\cos(6\pi z),$$
$$\left.\frac{1}{4}\sin(4\pi z)+\frac{1}{2}+\frac{1}{8}\sin(6\pi z)+\frac{1}{8}\sin(2\pi z)-\frac{1}{8}\cos(2\pi z)+\frac{1}{8}\cos(6\pi z)\right]$$

$$\left.\left[\frac{-1}{2}, \frac{-1}{2}, \frac{-1}{2}, \frac{1}{2}\right], z\right), \text{UNAPPLY}\left(\begin{bmatrix}0\\0\\0\\0\end{bmatrix}, z\right), \text{intervallpartition}(0 .. 1, 256)\right)$$

3394.992

```
> plotsetup(window):
  display(film1a,insequence=true,title="Film 1a: Wanderung längs
  <1,1,1,1>");
  plotsetup(default):
> plotsetup(window):
  display(film6a,insequence=true,title="Film 6a: Drehung in
  x',y'");
  plotsetup(default):
> plotsetup(window):
  display(film7a,insequence=true,title="Film 7a: Drehung in x',y'
```

```
  mit 1/3 Verschiebung nach außen");
  plotsetup(default):
> plotsetup(window):
  display(film8a,insequence=true,title="Film 8a: Drehung
  alpha=beta=gamma=z");
  plotsetup(default):
> if false then
    mkdir("4D-Würfel-via-convex-film1a-pov"):
    for i to nops(op(1,film1a)) do

  plotsetup(pov,plotoptions="phi=45,theta=45",plotoutput=sprintf("
  4D-Würfel-via-convex-film1a-pov/%03d.pov",i));
      print(display(op([1,i],film1a)));
    end do;
    plotsetupt(default):
  end if:
> if false then
    mkdir("4D-Würfel-via-convex-film6a-pov"):
    for i to nops(op(1,film6a)) do

  plotsetup(pov,plotoptions="phi=45,theta=45",plotoutput=sprintf("
  4D-Würfel-via-convex-film6a-pov/%03d.pov",i));
      print(display(op([1,i],film6a)));
    end do;
    plotsetupt(default):
  end if:
> if false then
    mkdir("4D-Würfel-via-convex-film7a-pov"):
    for i to nops(op(1,film7a)) do

  plotsetup(pov,plotoptions="phi=45,theta=45",plotoutput=sprintf("
  4D-Würfel-via-convex-film7a-pov/%03d.pov",i));
      print(display(op([1,i],film7a)));
    end do;
    plotsetupt(default):
  end if:
> if false then
    mkdir("4D-Würfel-via-convex-film8a-pov"):
    for i to nops(op(1,film8a)) do

  plotsetup(pov,plotoptions="phi=45,theta=45",plotoutput=sprintf("
  4D-Würfel-via-convex-film8a-pov/%03d.pov",i));
      print(display(op([1,i],film8a)));
    end do;
    plotsetupt(default):
  end if:
>
```